Chapter-3

- Wheel Alignment
- Wheel Kinematics and Compliance
- Steering Performance Criteria for Handling
Components of Suspension

- Linkage
- Bearings, Bushings
- Springs
- Dampers
Wheel Geometry
Wheel Geometry

The brake caliper is often used as a design element in exotic sports cars. It is located at the opposite side of the spindle to the steering arm. It limits the spoke design on the front wheels. More discounting can be achieved by using a wider rim or increasing the scrub radius, but these may be detrimental to performance and handling.

The spoke thickness is determined by structural requirements.
Wheel Geometry
Wheel Alignment

• A wheel Alignment is the adjustment of the suspension and steering to ensure proper vehicle handling with minimum tire wear.

• A change in alignment angles may result from one or more of the following factors:
  – Wear of the steering and suspension components
  – Bent or damaged steering and suspension parts
  – Sagging springs, which can change the ride height of the vehicle and therefore the alignment angles.
Alignment- Related Problems

• Pull
  – A pull is generally defined as a definite “tug” on the steering wheel toward the left or the right while driving straight on a level road

• Wander
  – A wander is a condition in which almost constant steering wheel corrections by the driver are necessary to maintain a straight-ahead direction on a straight level road
Camber

- Camber is the inward or outward tilt of the wheels from true vertical as viewed from the front or rear of the vehicle.
Camber

- If the top of the tire is tilted out, then camber is positive
- If the top of the tire is tilted in, then camber is negative
- If the tilt of the wheel is truly vertical, camber is zero
- Camber is measured in degrees or fraction of degrees
- Camber can cause tire wear if not correct
  - Excessive positive camber causes scuffing and wear on the outside edge of the tire
  - Excessive negative camber causes scuffing and wear on the inside edge of the tire
Camber

- Incorrect camber can cause excessive wear on wheel bearings. Many vehicle manufacturers specify positive camber so that the vehicle weight is applied to the larger inner wheel bearing and spindle. As the vehicle is loaded or when the spring sag, camber usually decreases. If camber is kept positive, then the running camber is kept near zero degrees for best tire life.
Camber

- Camber can cause pull (Camber Thrust) if it is unequal side to side. The vehicle will pull toward the side with the most positive (or least negative) camber. A difference of more than half a degree from one side to the other will cause the vehicle to pull
Camber Thrust

The lateral force that arises due to an inclination of the tyre from the vertical is referred to as camber thrust.

Generation of lateral force due to camber angle

Generation of lateral force due to camber angle
Camber Changes

Positive Cambered Wheels move inward when loaded

Vertical Load
Camber Change

Lateral Load Transfer and compression of Suspension, hence change in Camber

Upper Control Arm

Lower Control Arm

Instantaneous Centre (kinematic)
Camber Changes
Camber Change Effect in Cornering

Vehicle Having Positive Camber
Camber Changes Happen
Vehicle Understeers

\[ \frac{mV^2}{R} \]

Centrifugal Force
Camber Thrust
Lateral Force
Camber Change Effect in Cornering

Vehicle Having Negative Camber

Camber Changes Happen

Vehicle Oversteers
Suspension Travel
Parallel Wheel Travel

Bump Travel

– Vertical distance wheel is able to move up from static position, with reference to vehicles sprung mass

Rebound Travel

– Vertical distance wheel is able to move down from static position with reference to vehicles sprung mass
Camber Change
Parallel Opposite Wheel Travel
Steering Angle and Bump Steer
Camber Change in Solid Axle Suspension

Solid axle suspension characteristics: Camber change on bumps, none on rebound, large unsprung weight
Camber

- Camber is not adjustable on many vehicles
- If camber is adjustable, the change is made by moving the upper or the lower control arm or strut assembly by means of one of the following methods
  - Shims
  - Eccentric Cams
  - Slots
- Camber should be equal on both sides; however, if camber cannot be adjusted exactly equal, make certain that there is more camber on the front of the left side to help compensate for the road crown (half a degree maximum difference) in LHD, opposite for RHD
Toe

- Toe is the difference in distance between the front and rear of the tires

Toe-in (above), toe-out (below):
Most vehicle manufacturers specify a slight amount of toe-in to compensate for the natural tendency of the front wheels to spread apart (become toed-out) due to the centrifugal force of the rolling wheels acting on the steering linkage-rear wheel driven.

Some manufacturers of front wheel drive vehicles specify a toe-out setting to compensate for the toe-in forces created by the engine drive forces on the front wheels.

Normal wear to the tie rod ends and other steering linkage parts usually causes toe-out.
Toe

- Front wheel Drive

Drive Axles Toe in during running
Toe

- Toe is measured in fractions of degrees or in fractions of an inch (usually sixteenths), millimeters (mm), or decimals of an inch (such as 0.06”)
- Incorrect toe is the major cause of excessive tire wear
- Toe causes camber-type wear on one side of the tire if not correct
Toe

- Incorrect front toe does not cause a pull condition. Incorrect toe on the front wheels is split equally as the vehicle is driven because the forces acting on the tires are exerted through the tie rod and steering linkages to both wheels.
- Incorrect (unequal) rear toe can cause tire wear. If the toe of the rear wheels is not equal, the steering wheel will not be straight and will pull toward the side with the most toe-in.
Toe

- Front toe adjustment must be made by adjusting the tie rod sleeves correctly
Caster

- Caster is the forward or rearward tilt of the steering axis in reference to a vertical line as viewed from the side of the vehicle. Steering axis is defined as the line drawn through the upper and lower steering pivot points.
- On an SLA suspension system, the upper pivot is the upper ball joint and the lower pivot is the lower ball joint. On a MacPherson strut system, the upper pivot is the centre of the upper bearing mount and the lower pivot point is the lower ball joint.
- Zero Center means that the steering axis is straight up and down, also called zero degrees or perfectly vertical.
Caster

- Positive caster is present when the upper suspension pivot point is behind the lower pivot point (ball joint) as viewed from the side.
- Negative caster is present when the upper suspension pivot point is ahead of the lower pivot point (ball joint) as viewed from the side.
- Caster is measured in degrees of fractions of degrees.
Caster- Camber Roll

- Caster is not a tire wearing angle, but positive caster does cause changes in camber during a turn. This condition is called camber roll.
Caster

- Caster is a stability angle
  - If caster is set positive, vehicle steering will be very stable (will tend to be straight with little steering wheel correction needed) and help with steering wheel
  - If the caster is positive, the steering effort will increase with increasing positive caster. Greater road shocks will be felt by the driver when driving over rough road surfaces. Vehicles with as high as eleven degrees of positive caster usually use a steering dampener to control possible shimmy at high speeds and to dampen the snap-back of the spindle after a turn
  - If caster is negative, or excessively unequal, the vehicle will not be as stable and will tend to wander. If a vehicle is heavily loaded in the rear, caster increase as shown
Caster Changes

This movement may be due to vehicle inertia

Assume wheel is locked up

Caster Changes leads to camber change hence camber roll
Caster Change (Parallel Opposite)
Caster angle v/s Wheel Travel (Parallel)
Caster

- Caster could cause pull if unequal. The vehicle will pull toward the side with least positive caster.
- Caster is not adjustable on many vehicles.
- If caster is adjustable, the change is made by moving either the lower or the upper pivot point forward or backward by means of one of the following methods:
  - Shims
  - Eccentric Cams
  - Slots
  - Strut rods
Caster Change and Aligning Moment

Centrifugal Force

Lateral/Grip Force

Tyre Slip Angle

Positive Caster point

Side force

Aligning Moment
Caster Change and Aligning Moment

Tyre Slip Angle

Positive Caster point

Aligning Moment

Negative caster
Steering Axis Inclination (SAI)

- The steering axis is the angle formed between true vertical and an imaginary line drawn between the upper and lower pivot points of the spindle.
- Steering axis inclination (SAI) is the inward tilt of the steering axis. SAI is also known as KPI and is the imaginary line drawn through the kingpin as viewed from the front.
Steering Axis Inclination (SAI)

- The front view axis inclination angle adds steering returnability by lifting the front axle in a turn.

When the wheel is turned, you recognise the lifting of the vehicle (on the ball). If you press the ball, the turned wheel immediately goes into the straight ahead position.
Steering Axis Inclination (SAI)
Steering Geometry
Scrub Radius
Effect of Scrub Radius on Steering Due to Road Disturbance

Positive Scrub Radius
Disturbance Creates outboard moment

Negative Scrub Radius
Disturbance Creates inboard moment

Rear Wheel Driven
Effect of Scrub Radius on Steering Due to Road Disturbance

Positive Scrub Radius
Traction Force Creates inboard moment
Traction is greater than Road Disturbance

Negative Scrub Radius
Traction Force Creates outboard moment
Effect of Scrub Radius During Braking

Positive scrub radius will cause the vehicle to veer towards the side with the greater effort. Negative scrub radius will cause the vehicle to veer away from the side of greatest effort. How much it veers depends on the size of the scrub radius.

During braking, on any type of drive, if braking effort is greater on one side of the vehicle than the other, positive scrub radius will cause the vehicle to veer towards the side with the greater effort. Negative scrub radius will cause the vehicle to veer away from the side of greatest effort. How much it veers depends on the size of the scrub radius.
Scrub Radius and Diagonal Split Brake

- Vehicles with a diagonal-split brake system have negative scrub radius built into the steering geometry. If one half of the brake system fails, then the vehicle will tend to pull up in a straight line.

![Diagram showing scrub radius and diagonal split brake](image)

- More brake effort produces a couple as shown.
- Reaction due to braking produces a clockwise couple.
- The braking force tries to turn counterclockwise.
Scrub Radius Change
Scrub Radius v/s Wheel Travel

The graph shows the relationship between scrub radius and wheel travel. The scrub radius decreases as the wheel travel increases. The blue line represents the scrub radius for the left wheel, and the red line represents the scrub radius for the right wheel.
Included Angle

- The included angle is the SAI added to the camber reading of the front wheel only. Included angle is an important angle to measure when diagnosing vehicle handling or tire wear problems.
Steering Knuckle
Wheel angles
## Alignment Specifications at Curb Height

<table>
<thead>
<tr>
<th>Front Wheel Alignment</th>
<th>Acceptable Alignment Range at Curb Height</th>
<th>Preferred Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camber - All*</td>
<td>-0.6° to +0.6°</td>
<td>+0.0°</td>
</tr>
<tr>
<td>*Side To Side Differential</td>
<td>0.7° or less</td>
<td>0.0°</td>
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<tr>
<td>Total Toe All Vehicles (See Note) Specified In Degrees</td>
<td>0.4° In to 0.0°</td>
<td>0.2° In</td>
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<tr>
<td>Caster*</td>
<td>Reference Angle</td>
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<tr>
<td>All Models</td>
<td>+2.0° to +4.0°</td>
<td>+3.0°</td>
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<tr>
<td>*Side To Side Caster Differential Not to Exceed</td>
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<td>0.0°</td>
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<table>
<thead>
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<th>Rear Wheel Alignment</th>
<th>Acceptable Alignment Range at Curb Height</th>
<th>Preferred Setting</th>
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<td>-0.1°</td>
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<tr>
<td>Total Toe* All Vehicles (See Note) Specified In Degrees</td>
<td>0.2° Out to 0.4° In</td>
<td>0.1° In</td>
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<tr>
<td>Thrust Angle</td>
<td>-0.15° to +0.15°</td>
<td></td>
</tr>
</tbody>
</table>

*TOE OUT When Backed On Alignment Rack Is TOE IN When Driving.

**NOTE:** Total toe is the arithmetic sum of the left and right wheel toe settings. Positive is Toe-in, negative is Toe-out. Total Toe must be equally split between each front wheel to ensure a centered steering wheel. Left and Right toe must be equal to within 0.02 degrees.
Suspension Kinematics

- A basic characteristic of suspension system is the change in orientation and position of the wheel under the wheel stroke, which is called kinematic characteristic and it strongly influences the handling and the stability of the vehicle.
- Kinematic design of a suspension system involves determining the positions of hardpoints or kinematic design points.
- Suspension design factors such as toe, camber and caster are decided by the location of hardpoints.
Compliance

- Compliance is deliberately introduced into the suspension systems through bushings to achieve good ride and handling.
- Bushings are rubber members provided in suspension and steering sub-systems to avoid metal-to-metal friction during kinematic motion.
- Two types of compliance are of interest – lateral and longitudinal force compliance.
- Specific Bushings are required to have desirable stiffness in specific orientations to meet compliance
Bush Stiffness

- *For MacPherson Suspension*
- $Y$ stiffness of lca_front and lca_rear bushings affects toe and camber in braking and driving
- $X$ stiffness of rack_house bushing (steering sub-system) affects toe and camber in braking and driving
- $Z$ stiffness of rack_house bushing (steering sub-system) affects toe and camber under lateral forces
Elastomer/Rubber Bushes

Suspension Bushes
Automobile Suspension
Kinematics and Compliance Test Rig with Test Vehicle
# Suspension Bushes

<table>
<thead>
<tr>
<th>Type</th>
<th>Applications</th>
</tr>
</thead>
</table>
| Single Bonded Bushes | • Damper bushes  
• Engine torque rods  
• Low cost suspension arms |
| Double Bonded Bushes | • Damper bushes  
• Suspension arms, where there is insufficient support for a single bonded bush |
| Interleaved Bushes  | • High articulation positions  
• Multi-link sports suspensions |
| Hydraulic Bushes    | • Front suspension compliance bushes  
• Rear suspension trailing arm bushes  
• Subframe mountings |

- **Single Bonded Bushes**: Provides a low cost pivot.
- **Double Bonded Bushes**: Provides a controlled stiffness pivot.
- **Interleaved Bushes**: Provides a controlled stiffness pivot with low torsional stiffness.
- **Hydraulic Bushes**: Provides damping control.
Ackermann and Centre Point Steering

Low lateral Acceleration

The intention of Ackermann geometry is to avoid the need for tyres to slip sideways when following the path around a curve. The geometrical solution to this is for all wheels to have their axles arranged as radii of a circle with a common centre point. As the rear wheels are fixed, this centre point must be on a line extended from the rear axle. Intersecting the axes of the front wheels on this line as well requires that the inside front wheel is turned, when steering, through a greater angle than the outside wheel.
Steering System Performance

\[
\delta_o = \tan^{-1} \left( \frac{L}{R + t/2} \right) \approx \frac{L}{R + t/2}
\]

\[
\delta_i = \tan^{-1} \left( \frac{L}{R - t/2} \right) \approx \frac{L}{R - t/2}
\]
A simple approximation to perfect Ackermann steering geometry may be generated by moving the steering pivot points inward so as to lie on a line drawn between the steering kingpins and the centre of the rear axle. The steering pivot points are joined by a rigid bar called the tie rod which can also be part of the steering mechanism, in the form of a rack and pinion for instance. With perfect Ackermann, at any angle of steering, the centre point of all of the circles traced by all wheels will lie at a common point. Note that this may be difficult to arrange in practice with simple linkages, and designers are advised to draw or analyze their steering systems over the full range of steering angles.
The kinematic geometry of the relay linkages and steering arms is usually not a parallelogram which would produce equal left and right steer angles, but rather a trapezoidal to more closely approximate” Ackermann geometry which steers the inside wheel to a greater angle than outside wheel. Interference with the wheel usually prevents design for good Ackermann.

Proper design of the Ackermann geometry is a function of the vehicle wheel base and tread.
Modern cars do not use *pure* Ackermann steering, partly because it ignores important dynamic and compliant effects, but the principle is sound for low speed manoeuvres.

### Steering Knuckle Angle

**Table:**

<table>
<thead>
<tr>
<th>Wheelbase (inches)</th>
<th>Tread (inches)</th>
<th>Angle X</th>
<th>Wheelbase (inches)</th>
<th>Tread (inches)</th>
<th>Angle X</th>
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<td>42.5</td>
<td>72 degrees</td>
<td>100 90</td>
<td>60</td>
<td>66 degrees</td>
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<tr>
<td>80 70</td>
<td>34</td>
<td></td>
<td>80 70</td>
<td>48</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>100 90</td>
<td>54</td>
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<td></td>
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<td>80 70</td>
<td>42</td>
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<tr>
<td>100 90</td>
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<td>71 degrees</td>
<td>100 90</td>
<td>56</td>
<td>65 degrees</td>
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<tr>
<td>80 70</td>
<td>36</td>
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<td>80 70</td>
<td>50</td>
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<td>45</td>
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<tr>
<td></td>
<td>33.5</td>
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</table>

Since the results of most calculations must be graphically verified, one could use Mr. Korff's table as a starting point, then adjust the angles to remove real-world errors.
Steering Systems

Rack and Pinion Steering System
Basic Steering Systems

Recirculating Ball Steering System
Steering System Video

Video
Steering System – Hydraulic Power
Electric Power Assisted Steering
Steering Geometry Error

• Steering actions that arise from suspension motions are known as steering geometry error
• The errors are: bump steer (ride steer) and roll steer
• It is essential to run zero bump steer: if the wheel steers when it runs over a bump or when the car roll in a turn, the car will travel on a path that the driver did not intend
Bump (Ride) Steer

- Bump Steer
Bump(Ride) Steer Equipment

Adjust Front Bump Steer by:
1 - Shimming outboard track rod end.
2 - Moving rack or end clevis vertically.

The Rolling Radius determines ride height point for indicators.
Zero Ride Steer-Desirable

Outer Tie Rod Path developed by the suspension for "0" toe change.

Center of Arc, or position of inner tie rod. It's the path the outer should follow to have no toe change as the wheel moves up and down.

(REAR VIEW OF LEFT FRONT WHEEL)

A plot of toe-in measured at front and rear of a scribed wheel for up and down motion of the wheels. This configuration is called zero toe steer and is generally desirable.

The straightness of the line indicates the tie rod length. This curve shows correct length.
Nonlinear ride steer-tie rod length incorrect

This picture shows the actual tie rod to be too short. Here the wheel will toe-out in jounce and rebound both, and at normal car height will pass through the initial toe setting.

Curvature of the pattern and direction of convexity indicates tie rod length. The curve at the right indicates the tie rod is too short, but height is correct.
Roll Steer Behaviour Experimentally Measured on a Vehicle

![Graph showing roll steer behaviour](image)
Steering System Performance Measures

- The specific design of a steering system geometry has a well-recognized influence on steering performance measures such as
  - Center feel
  - Steering Returnability
  - Steering Ratio
  - Steering ratio to cornering
  - Steering ratio to braking
  - Steering efforts
Steering Centre Feel

• Centre Feel
Steering Returnability-Alignment Moment

- When the steering wheel is released, the wheels must return automatically to the straight-running position and must remain stable in this position.

- Tyre Slip Angle $\alpha$
Tyre Slip Angle
Steering Kinematics

- Steering kinematics and axle design must be such that, although the driver receives feedback on the adhesion between wheels and road surface, the steering wheel is not subjected to any forces from the spring motion of the wheels or from motive forces (front wheel drive)
  - Steering – Axis inclination causes the front section of the vehicle to lift when the wheels are at an angle. This leads to a caster dependent on the steering angle
  - Toe-in (toe-out) is a slip angle present even during straight running travel. This tensions the linkages and causes a rapid build-up of transverse forces when the wheels are at an angle
Steering Kinematics

- Caster produces a lever arm for side forces, i.e. speed dependent return torque (Alignment Torque)
- Kingpin offset determines the extent to which the steering system is affected by interference factors: brakes pulling unevenly, motive forces under traction/overrun in front wheel drive vehicles. In modern designs, the aim is to achieve a steering offset (Scrub Radius/Pivot Radius) which is zero to slightly negative
Caster Change and Aligning Moment

- Positive Caster point
- Aligning Moment
- Tyre Slip Angle
- Side force
- Centrifugal Force
- Lateral/Grip Force

Lateral Slip Angle
Caster Change and Aligning Moment

Tyre Slip Angle

Positive Caster

Positive Caster point

Aligning Moment

Negative Caster
Self Aligning Moment

\[ M_{AT} = (M_{Zl} + M_{Zr}) \cos \sqrt{\lambda^2 + \nu^2} \]
Aligning Moment

Pneumatic Trail

$F_y$
$M_z$

Direction of Heading

Slip Region
Contact Patch

Slip Angle, $\alpha$

Direction of Travel
Self Aligning Moment or Torque

Centrifugal Force

Side Force

Self Aligning Torque

Castor line point
Steering Ratio

- The steering ratio is defined as the ratio of steering wheel rotation angle to steer angle at the road wheels.
- Normally these range from 15 or 20 to 1 on passenger cars, and 20-36 to 1 on trucks.
- Steering ratio allow for easy steering of the front wheels. Low ratios such as 12:1 give quick but stiff steering where as high ratios such as 20:1 provide slow but easier steering.
- Because of the compliance, with increasing steer angles, the actual steering ratio will be much more than designed ratio.
Steering Ratio

Because of the compliance and steer torque gradients with increasing steer angles, the actual steering ratio may be as much as twice the designed ratio. Fig shows experimental measurements on a truck which illustrate the phenomenon.
Understeer Gradient Measured at the Steering wheel and Road Wheel of a Truck
Understeer Gradient due to Steering

\[ K_{strg} = \frac{W_f (r \nu + p)}{K_{ss}} \]

where:

- \( K_{strg} \) = Understeer increment (deg/g) due to steering system
- \( W_f \) = Front wheel load (lb)
- \( r \) = Wheel radius (in)
- \( p \) = Pneumatic trail associated with aligning torque (in)
- \( \nu \) = Caster angle (rad)
- \( K_{ss} \) = Steering stiffness (in-lb/deg) between road wheel and steering wheel
Steering Ratio for Cornering

\[ \delta = 57.3 \frac{L}{R} + K \left( \frac{V^2}{gR} \right) \]

At high speeds for an understeered vehicle steering angle increases, for cornering at high speed, it is good to have smaller steering ratios
Active Steering

• Need for Active Steering
  – When vehicle Yaws during Braking due to imbalanced braking forces
  – Vehicles yaws due to split Mu
  – Vehicle Aligning moment changes its direction due to caster change
• Steering Ratio should be small to reduce the yaw rate by steering in the opposite direction quickly
• Normal speeds and normal conditions, steering torque rate should not have steep gradient-reasonable steering ratio
• Hence, need for variable steering ratio
Active Steering Concept

• When driving at lower speeds - such as in city traffic, when parking or on winding mountain roads, Active Steering increases the size of the steering angle- Low steering ratio
• At medium speeds, steering is easier
• To ensure smoothness at higher speeds, as of around 120 to 140 km/h Active Steering becomes more indirect. Active Steering therefore reduces the amount of change in the steering angle for every movement of the steering wheel. This gives the driver the advantage of more precise steering at higher speeds, and ensures great stability and more comfort
• If the vehicle is threatened with instability, such as by oversteering or braking on a changeable surface, Active Steering helps to overcome it. For example, in order to reduce unsafe yaw, Active Steering can increase the angle of steering wheels faster than even the most expert driver.
Active Steering Concept

At the heart of Active Steering system is the planetary gear set integrated into the steering column. An electric motor in the joint adjusts the front wheels' steering angle in proportion to the Sedan's current speed.
BMW Active Steering
Steering System Forces and Moments

- The ground reactions on the tire are described by three forces and moments, as follows:
  - Normal force
  - Tractive force
  - Lateral force
  - Aligning torque
  - Rolling resistance moment
  - Overturning moment

- On front-wheel-drive cars, an additional moment is imposed by the drive torque.
Lateral Force - $F_y$

Longitudinal Force - $F_x$: Traction Force

Normal Force $F_z$
Steering System Forces and Moments

- The forces and moments imposed on the steering system emanate from those generated at the tire-road interface.
Steering Wheel Torque

- The reaction in the steering system is described by the moment produced on the steer axis, which must be resisted to control the wheel steer angle.
- The sum of moments from the left and right wheels acting through the steering linkages with their associated ratios and efficiencies account for the steering-wheel torque feedback to the driver.
Estimation of Steering Forces and Moments

\[ M_v = \text{Total moment from left and right wheels} \]

\[ F_{zl}, F_{zr} = \text{Vertical load on left and right wheels} \]

\[ d = \text{Lateral offset at the ground} \]

\[ \lambda = \text{Lateral Inclination Angle (KPI)} \]

\[ \delta = \text{Steer Angle} \]

\[ \nu = \text{Caster Angle} \]
Steering Forces and Moments

- The moment arising from vertical force acting

\[ M_v = -(F_{zl} + F_{zr})d \sin \lambda \sin \delta + (F_{zl} - F_{zr})d \sin \nu \cos \delta \]

- The moment arising from lateral force

\[ M_L = (F_{yl} + F_{yr})r \tan \nu \]

- The moment arising from traction force

\[ M_T = (F_{Xl} - F_{Xr})d \]

- Aligning Torque

\[ M_{AT} = (M_{Zl} + M_{Zr}) \cos \sqrt{\lambda^2 + \nu^2} \]

- Rolling resistance and Overturning moments have second order effect and are neglected
Steering Torque high for positive caster and low for negative caster
Steering Torque

![Graph showing steering torque vs steering angle for different lateral inclination angles.](image)

- **Steering Torque (in-lb)**
- **Steering Angle (deg)**

Graph: Steering Torque from Lateral Inclination Angle

- 1" Offset
- 10° Inclination Angle

- Left Wheel (800 lb)
- Right Wheel (600 lb)
- Total
Steering Torque

STEERING TORQUE FROM CASTER ANGLE

1" Offset
5° Caster Angle

Steering Torque (in-lb)

Left Wheel (800 lb)
Total
Right Wheel (600 lb)

Steering Angle (deg)
Moment about SA due to drive line Torque

- The torque in the driveline produces a moment about the steer axis.

\[ T_d = F_x r \]
\[ M_{SA} = F_x [d \cos \nu \cos \lambda + r \sin(\lambda + \zeta)] \]
The lateral inclination and caster are small enough that the cosine function can be assumed unity. Hence

\[ M_{SA} = F_X [d + r \sin(\lambda + \zeta)] \]

The forward force introduces a moment in the steering system which opposes the steer angle trying to steer the vehicle out of turn-make the vehicle understeer. That is hwy in Front wheel drive vehicles toe out is provided
Steering Effort

- From the foregoing equations compute the steering moments
- Estimate the Effort from the driver
- The difference should be the effort developed by the Assist
EPAS- Modelling and Analysis
Aims and Objectives

• The aim is to develop a controller which fulfills the two primary functions of an EPAS system
  – Reduce the amount of steering torque exerted by the driver
  – Control the return-to-centre motion of the steering wheel
Manually Operated - Rack and Pinion Steering System

- Hand Steering wheel
- Intermediate shaft
- Steering Knuckle
- Steering arm
- Wheel
- Steering column
- Steering shaft
- Tie rod joints
- Tie rod
- Rack
- Pinion
- Tie rod
Manually Operated - Rack and Pinion Steering System

Hand steering wheel: $\theta_{HW}$
Hand wheel torque: $T_{HW}$

Rack and Pinion Gear Ratio: $G_{PR}$

Torque applied to overcome the Road wheel torque load

Road wheel torque due to friction between Road wheel tyre and Road and Aligning moment acts as load

Road wheel steered angle: $\theta_{RW}$

Tactile and Visual feedback

All the torque required for steering the road wheel needs to be developed by the driver.
Motor Assisted-Rack and Pinion Steering System (EPAS)

Column Assisted Type
Steering Wheel Angle and Torque Sensor-EPAS

• The magnitude and direction of torque applied by the driver are sent by the torque sensor to the controller.
• The amount of torque required to steer the road wheel is calculated from vehicle speed which is received as a signal by the controller.
• The controller decides the assist torque to be developed by subtracting the hand wheel torque.
• The assist torque to be developed is divided by gear ratio between motor shaft and steering shaft to get the motor torque to be developed.
• Based on required motor torque, current value is calculated and supplied to the motor.
Motor Assisted-Rack and Pinion Steering System-EPAS

The total torque applied is the sum of Hand wheel torque and Motor Torque
EPAS - Detailed Modelling

Hand Wheel Torque applied- $T_{HW}$
Mass moment of inertia of Hand wheel with Steering Shaft- $J_{HW}$
Steering shaft torsional stiffness $k_{SS}$ and Damping Coefficient $c_{SS}$

Torque from Assist Motor: $T_{AM}$
Inertia of Gear and intermediate shaft System: $J_{GI}$
Intermediate shaft Stiffness $k_{IS}$ and Damping Coefficient $c_{IS}$
Pinion inertia- $J_p$
Pinion Shaft stiffness-$k_p$
Pinion damping-$c_p$

Mass of Rack- $M_R$
Mass of Tie Rods-$M_{LT}$, $M_{RT}$
Inertia of Steering Arm $J_{LSA}$, $J_{RSA}$
Wheel Inertia $J_{LW}$, $J_{RW}$
Forces on tyres $F_{LT}$, $F_{RT}$

C-Conversion from Rotary Linear or linear to Rotary motion
EPAS- Simplified Modelling

Total Torque

- Torque Applied by Driver
- Torque to be Developed by Motor

- Torque on Pinion Shaft
- Force to Torque Conversion
- Force on Rack
- Torque to Force Conversion
- Torque from Road wheel
Steering Torque about Steering Axis

Steering moments acting about Steer Axis are:
1. Steering Torque due to vertical forces: $M_V$
2. Steering Torque due to Lateral Forces: $M_L$
3. Steering Torque due to Traction Forces: $M_T$
4. Aligning Moment: $A_T$
5. Moment due to drive line torque for front wheel drive vehicle: $M_d$
Steering Torque from Road Wheel

\[ T_{\text{total}}(\text{About steering Axis}) = M_v + M_L + M_T + M_{\text{align}} + M_{\text{driveline}} + J_w \ddot{\delta}_w + c_w \dot{\delta} \]

\( J_w \) = Wheel inertia about SA axis
\( c_w \) = Tyre Damping
\( \ddot{\delta} \) = Tyre angular acceleration – zaxis
\( \dot{\delta} \) = Tyre angular velocity – zaxis
Estimation of Road Torque

\[ M_v = -(F_{zl} + F_{zr})d \sin \lambda \sin \delta + (F_{zl} - F_{zr})d \sin \nu \cos \delta \]

\[ M_L = (F_{yl} + F_{yr})r \tan \nu = (C_{\alpha l} \alpha_l + C_{\alpha r} \alpha_r)r \tan \nu \]

\[ M_T = (F_{xl} - F_{xr})d = \mu(F_{zl} - F_{zr})d \]

\[ M_{AT} = (M_{zl} + M_{zr}) \cos \sqrt{\lambda^2 + \nu^2} = (F_{yl} + F_{yr})(t_p + t_m) = (C_{\alpha l} \alpha_l + C_{\alpha r} \alpha_r)(t_p + t_m) \]

\[ M_d = F_x [d + r \sin(\lambda + \zeta)] = \mu(F_{zl} + F_{zr})[d + r \sin(\lambda + \zeta)] \]

\[ T = -(F_{zl} + F_{zr})d \sin \lambda \sin \delta + (F_{zl} - F_{zr})d \sin \nu \cos \delta + (C_{\alpha l} \alpha_l + C_{\alpha r} \alpha_r)r \tan \nu + \mu(F_{zl} - F_{zr})d + (C_{\alpha l} \alpha_l + C_{\alpha r} \alpha_r)(t_p + t_m) + \mu(F_{zl} + F_{zr})[d + r \sin(\lambda + \zeta)] \]

Fzl : Vertical load on left tyre
Fzr: vertical load on right tyre
FxL: Traction load on left
FxR: Traction load on right
Fx: Total Traction force
C\alpha : Cornering stiffness of tyre left and right
d : scrub radius
\delta: Road wheel steer angle
\alpha: tyre slip angle
\nu: castor angle
\lambda: Lateral inclination angle
\zeta: drive shaft angle
r : tyre radius
Estimation of Torque on Motor

\[ T_{total} = J_w \dddot{\delta} + c_w \ddot{\delta} - (F_{zl} + F_{zr})d \sin \lambda \sin \delta + (F_{zl} - F_{zr})d \sin \nu \cos \delta + (C_{cd} \alpha_l + C_{cr} \alpha_r) r \tan \nu + \mu(F_{zl} - F_{zr})d + (C_{cd} \alpha_l + C_{cr} \alpha_r)(t_p + t_m) + \mu(F_{zl} + F_{zr})[d + r \sin(\lambda + \zeta)] \]

\[ F_{TR} = \text{Force on tie Rod} = ( J_s \dot{\theta}_s + c_s \theta_s + k_s \theta_s + T_{total}) / L_{steering \ Arm} \]

\[ F_{Rack} = F_{TR} + ( M_{TR} \dot{x}_{TR} + c_{TR} x_{TR} + k_{TR} x_{TR} ) \]

\[ F_{pinion} = F_{Rack} + ( M_R \dot{x}_R + c_R x_R + kx_R ) \]

\[ T_{pinionshaf} = F_{pinion} R_{pinion} + J_p \dddot{\theta}_p + c_p \ddot{\theta}_p + k_p \theta_p + \text{anyothercomponent model} \]

\[ T_{final} = \frac{T_{pinion}}{G_p} = T_{handwheel} + T_{motor} \]
## Data Required For Simulation

<table>
<thead>
<tr>
<th>S. NO.</th>
<th>Parameter</th>
<th>Symbol</th>
<th>Range of Values</th>
<th>Unit</th>
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<td>Tyre Cornering Stiffness front left</td>
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<td>N/rad</td>
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<td>Castor angle</td>
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<td>16</td>
<td>Wheel steer Angle</td>
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<td>Deg or rad</td>
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## Data Required For Simulation

<table>
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<tr>
<th>S. No.</th>
<th>Parameter</th>
<th>Symbol</th>
<th>Values/Range</th>
<th>Unit</th>
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<td>Wheel inertia about SA</td>
<td>$J_w$</td>
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<td>Kg-m$^2$</td>
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<td>$c_p$</td>
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<td>Inertia,damping and stiffness</td>
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<td>Motor to Shaft Gear Ratio</td>
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<td>Motor current Vs Motor Torque Characteristics</td>
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<td>Motor Inductance</td>
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<tr>
<td>23</td>
<td>Motor Resistance</td>
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</tr>
</tbody>
</table>

Other wise find by mechanical geometric modelling
It includes intermediate shaft, rack and pinion, tie rod, steering arm.

$T_{\text{total}}$ from vehicle

It includes road wheel torque about steering axis which can be calculated if vehicle velocity is known.

$C_{\text{steerin}}$

Feed back in the form of speed, everything else has to be calculated in the controller.
\[ T_{\text{total \ Simulation}} \]

\[
\ddot{\delta} = -\left( \frac{c_w}{J_w} \right) \dot{\delta} + \left[ \frac{F_{zl} + F_{zr}}{J_w} \right] d \sin \lambda \sin \delta - \left[ \frac{F_{zl} - F_{zr}}{J_w} \right] d \sin \nu \cos \delta + \frac{T_{\text{total}}}{J_w} \\
+ \left[ \frac{C_{al} \alpha_l + C_{ar} \alpha_r}{J_w} \right] r \tan \nu + \mu \left[ \frac{F_{zl} - F_{zr}}{J_w} \right] d + \left[ \frac{C_{al} \alpha_l + C_{ar} \alpha_r}{J_w} \right] (t_p + t_m) + \\
\mu \left[ \frac{F_{zl} + F_{zr}}{J_w} \right] [d + r \sin(\lambda + \zeta)]
\]

\[
\delta = 57.3 \frac{L}{R} + \left[ \frac{W_f}{C_{af}} - \frac{W_r}{C_{ar}} \right] \frac{V^2}{gR} = 57.3 \frac{L}{R} + K \frac{V^2}{gR}
\]
Simulink Diagram

\[
\begin{align*}
T_{\text{Total}} &= \frac{1}{J_w} + \delta + \delta + \delta \\
\frac{C_w}{J_w} &+ \left[ \frac{F_{zl} + F_{zr}}{J_w} \right] d \sin \lambda \sin \delta - \left[ \frac{F_{zl} - F_{zr}}{J_w} \right] d \sin \nu \cos \delta \\
&+ \left[ \frac{C_{al} \alpha_l + C_{ar} \alpha_r}{J_w} \right] r \tan \nu + \frac{F_{zl} - F_{zr}}{J_w} d + \left[ \frac{C_{al} \alpha_l + C_{ar} \alpha_r}{J_w} \right] (t_p + t_m) + \mu \left[ \frac{F_{zl} + F_{zr}}{J_w} \right] [d + r \sin(\lambda + \zeta)]
\end{align*}
\]

Plot T vs \( \delta \)
Let us obtain these characteristics from ADAMS Modelling for specifications of the CAR.
<table>
<thead>
<tr>
<th>Vehicle Speed kmph</th>
<th>Turning Radius m</th>
<th>δ deg</th>
<th>Steering Torque T (Nm)</th>
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</table>
Control Algorithm

Torque Sensor → Hand Wheel Torque
Hand Wheel Torque → Steering Characteristics
Steering Characteristics → Assist Torque
Assist Torque → Current Vs Torque Characteristics
Current Vs Torque Characteristics → Current to Assist Motor

Vehicle speed → Steering angle & Direction
Steering angle & Direction → Steering Characteristics

Graph showing current vs torque characteristics at different vehicle speeds (10 kmph, 20 kmph, 30 kmph, 40 kmph, 50 kmph).